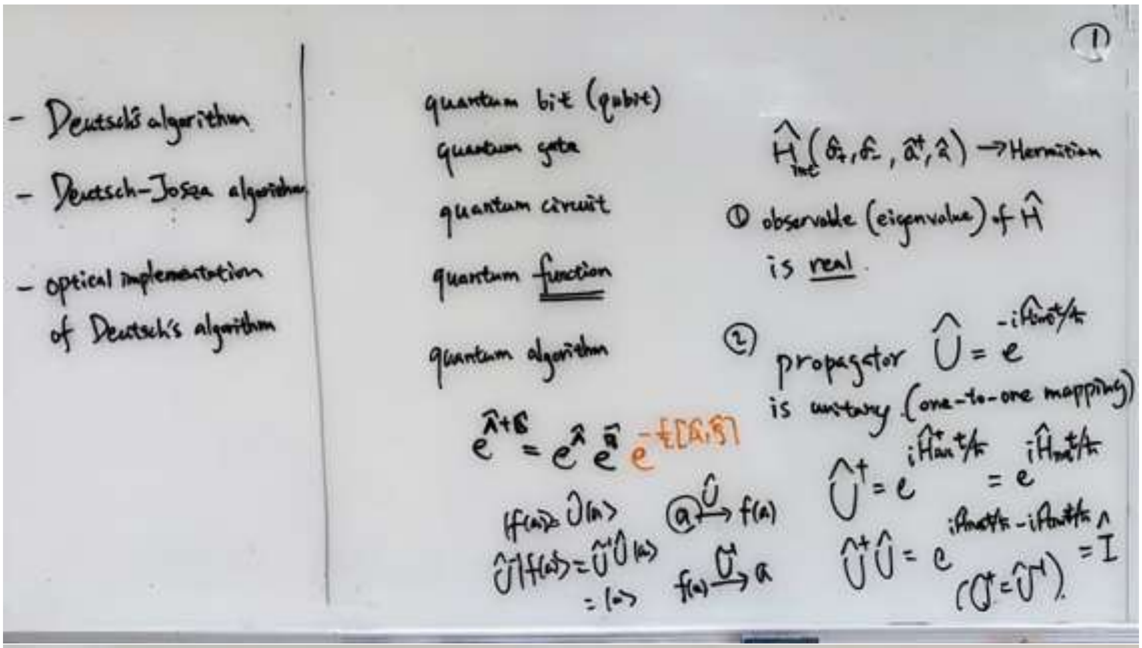
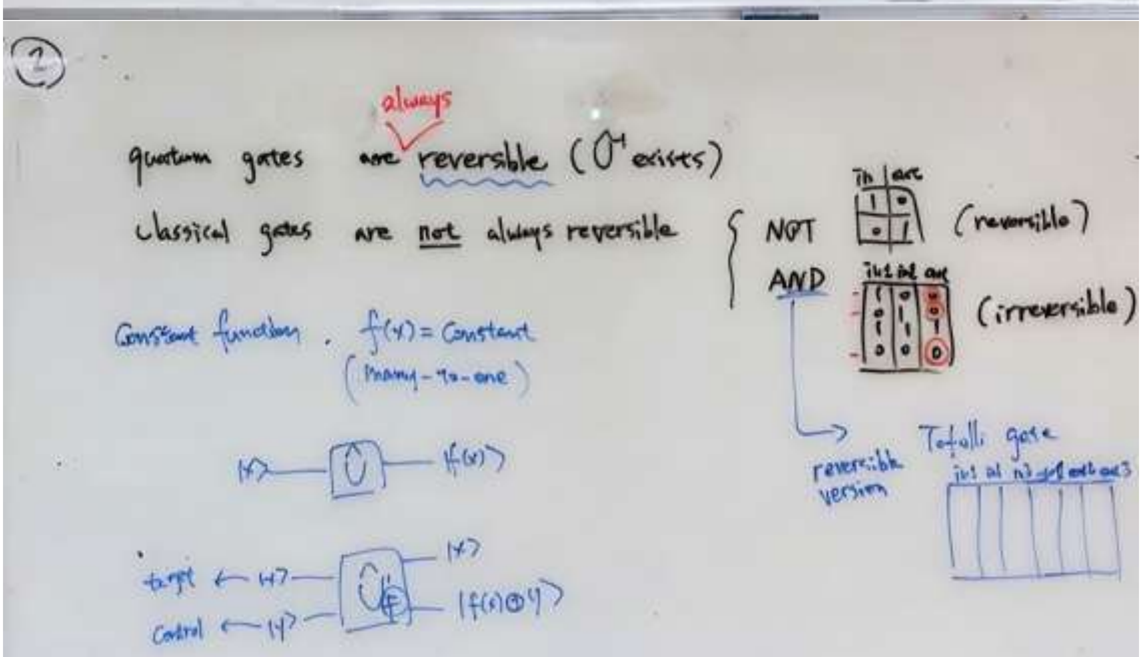
Lecture14 (spin) Deutsch algorithm and its optical implementation





③

- Deutsch's algorithm
- Deutsch-Josza algorithm
- optical implementation of Deutsch's algorithm

quantum algorithm — recipe → problem
(construction)

memory inquiry

$\{0.1\}^1$, $\{0.1\}^2 = \{00, 01, 10, 11\} \rightarrow 2^2 = 4$

$0 - 100 \rightarrow$ 6 classical bit
6 qubit

$\{0.1\}^6 \xrightarrow{6} 2^6 = 128$

$0 \rightarrow 0 \quad 0\cdot2^0 = 0$

$2 \rightarrow 10 \quad 1\cdot2^1 + 0\cdot2^0 = 2$

$5 \rightarrow 101 \quad 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0 = 5$

$10 \rightarrow 1010 \quad 1\cdot2^3 + 0\cdot2^2 + 1\cdot2^1 + 0\cdot2^0 = 10$

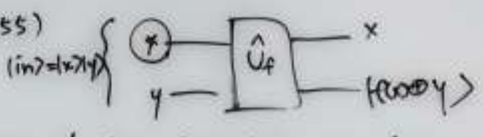
$64 \rightarrow 100000 \quad 1\cdot2^6 + 0\cdot2^5 + 0\cdot2^3 + 0\cdot2^2 + 0\cdot2^1 + 0\cdot2^0 = 64$

$100 \rightarrow 110100 \quad 1\cdot2^5 + 1\cdot2^4 + 0\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 0\cdot2^0 = 100$

④

* local property of the classical function $f(x)$

$\rightarrow f(89)$, $f(55)$



$|out\rangle = |x\rangle|f(x)\oplus y\rangle$

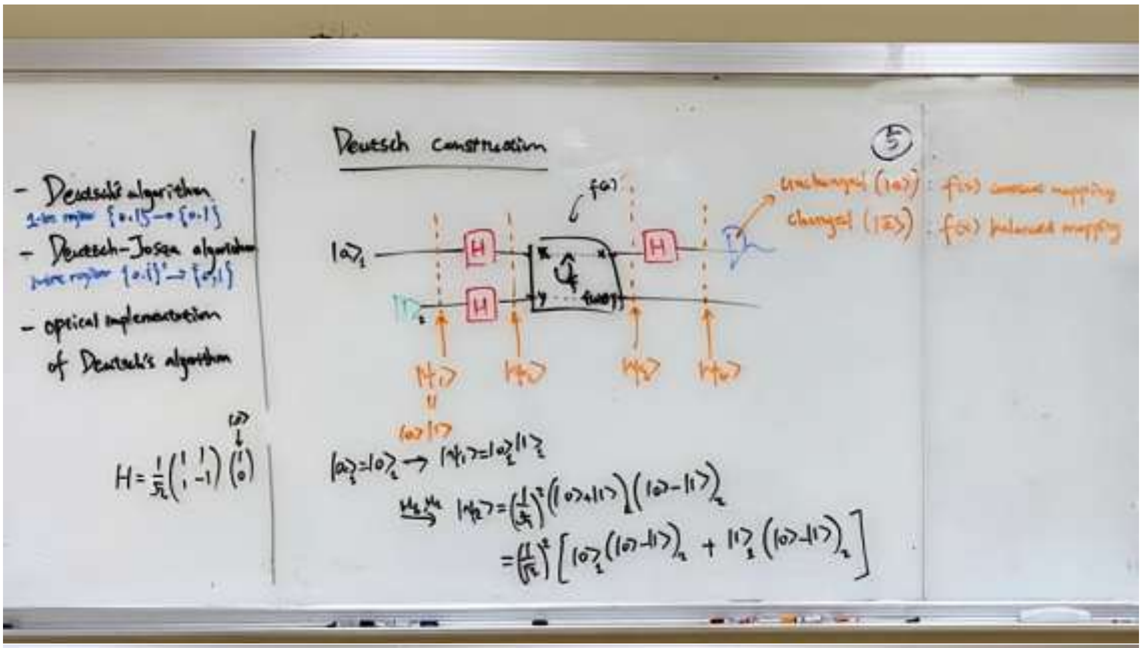
* global property of the classical function $f(x)$

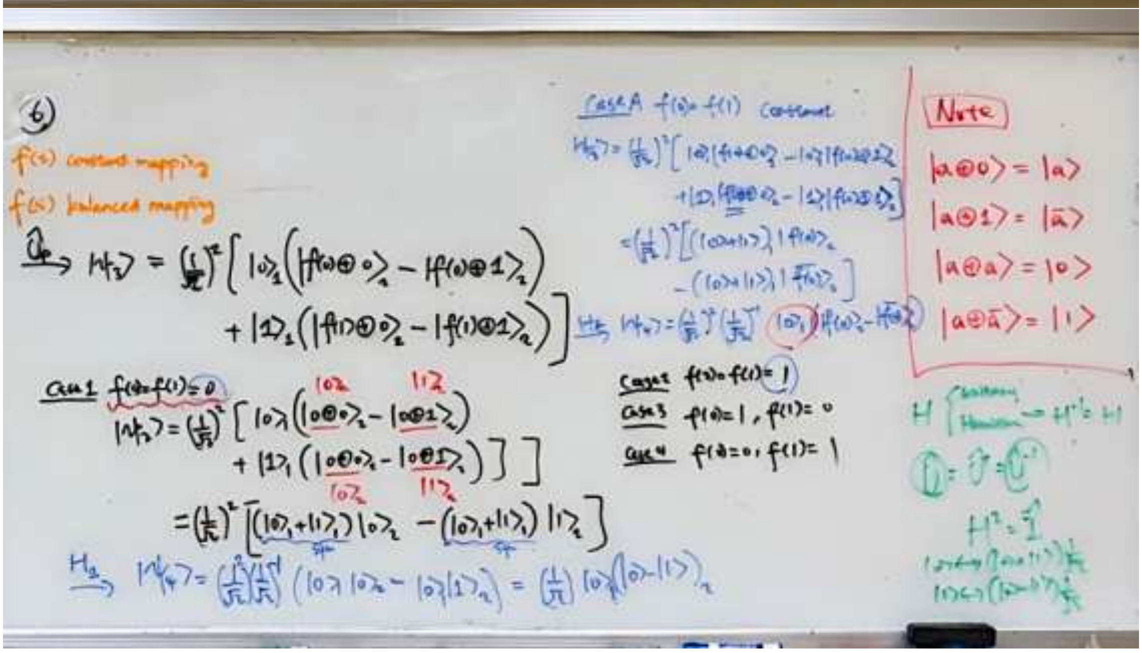
- constant mapping
- balanced mapping

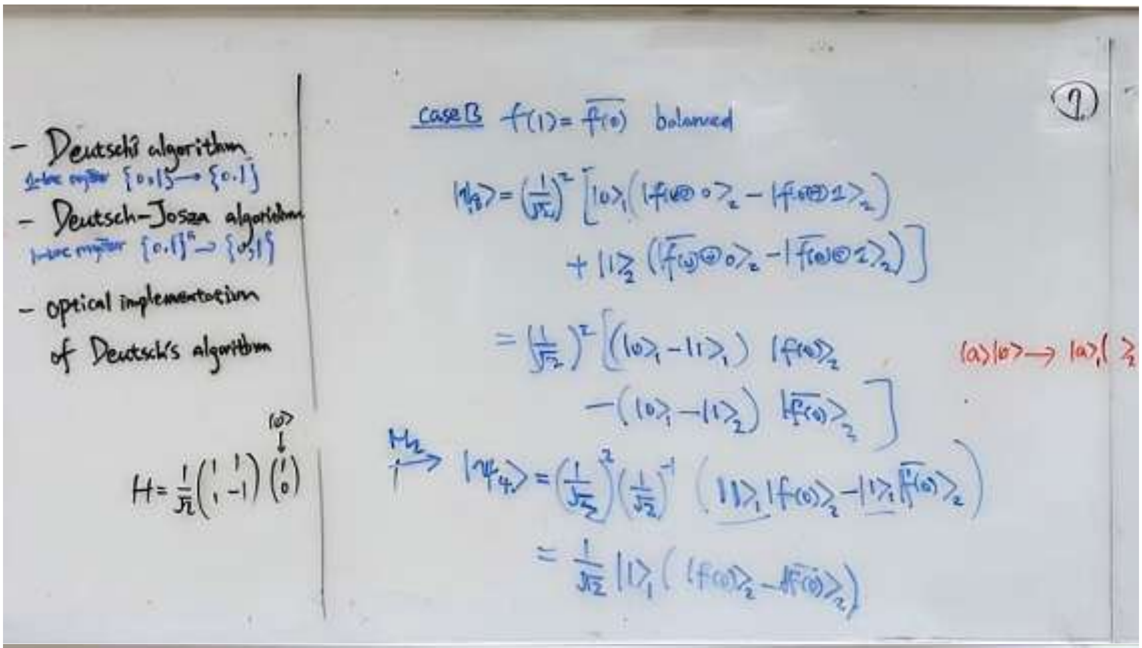
$\{0,1\}^6 \rightarrow \{0.1\}$

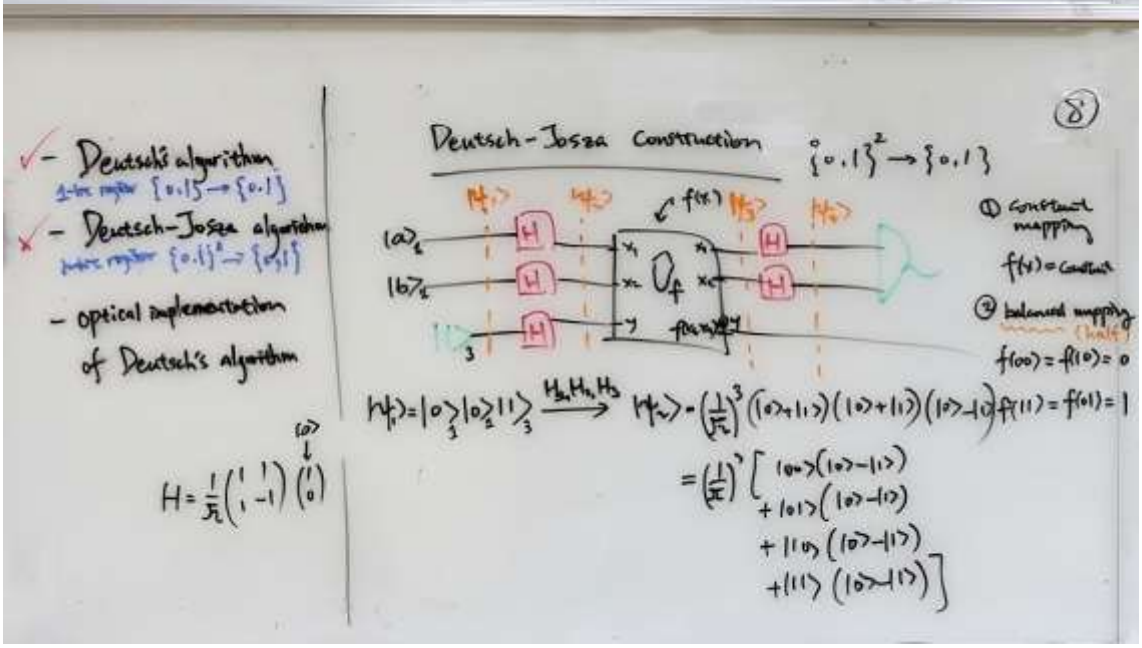
128 runs → classical computer

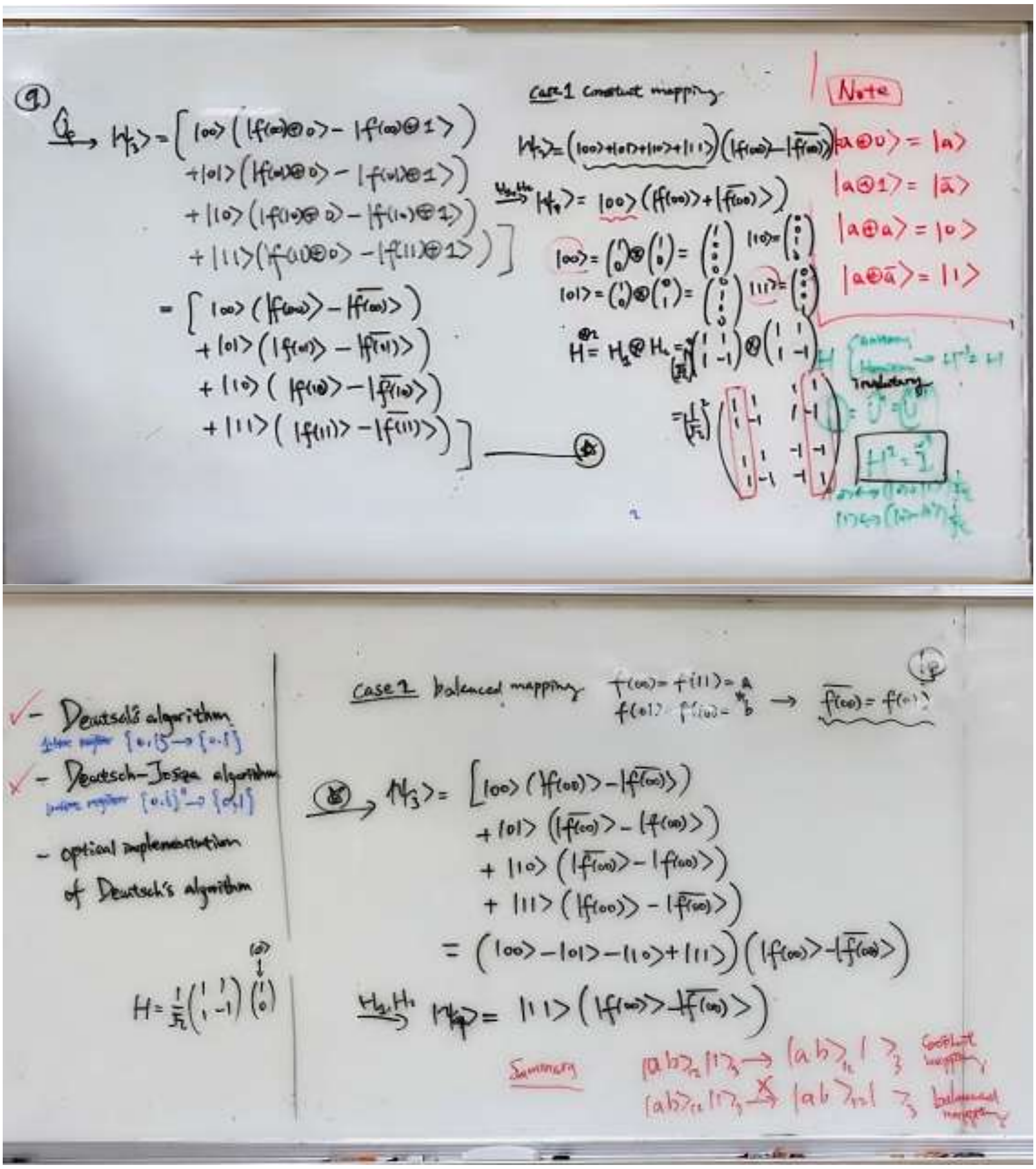
1 run → quantum computer (Deutsch-Josza algorithm)

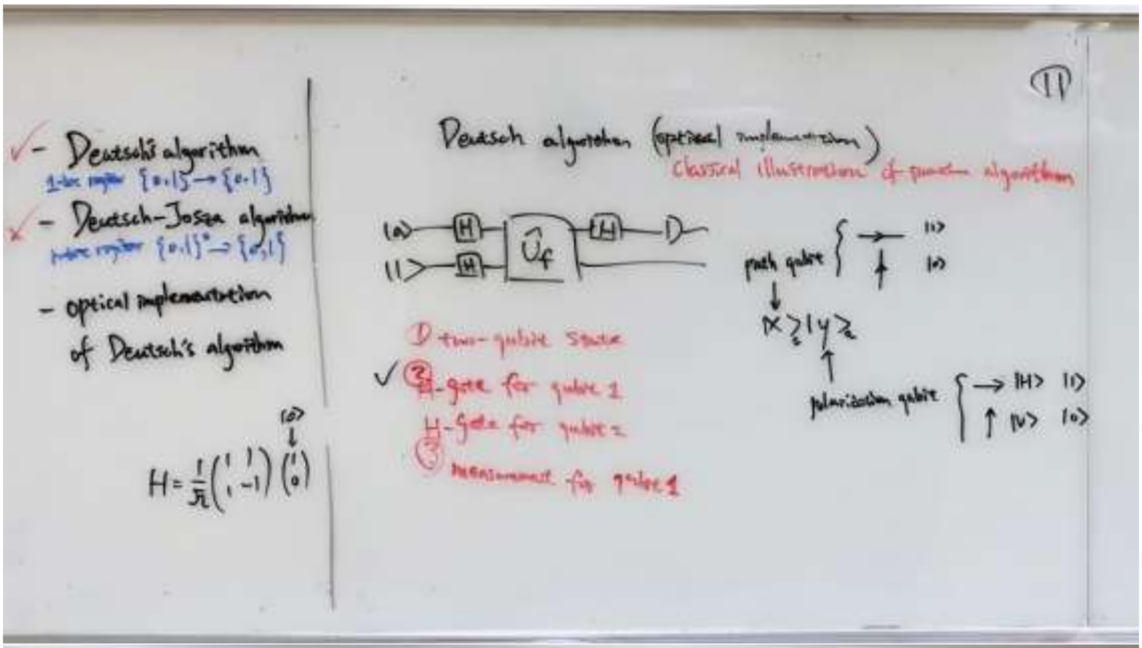

Deutsch construction
- Deutsch's algorithm
- Deutsch-Jozsa algorithm
- optical implementation of Deutsch's algorithm
f(x) constant mapping
f(x) balanced mapping

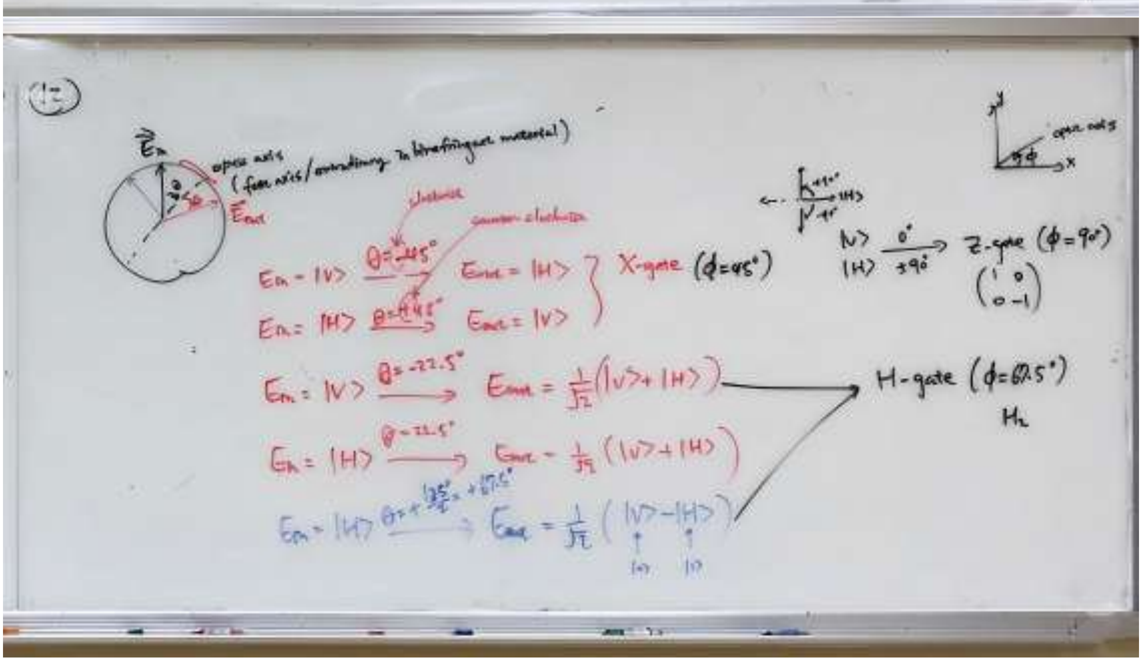

f(x) constant mapping
f(x) balanced mapping
Note
Case 1
Case 3
Case 4

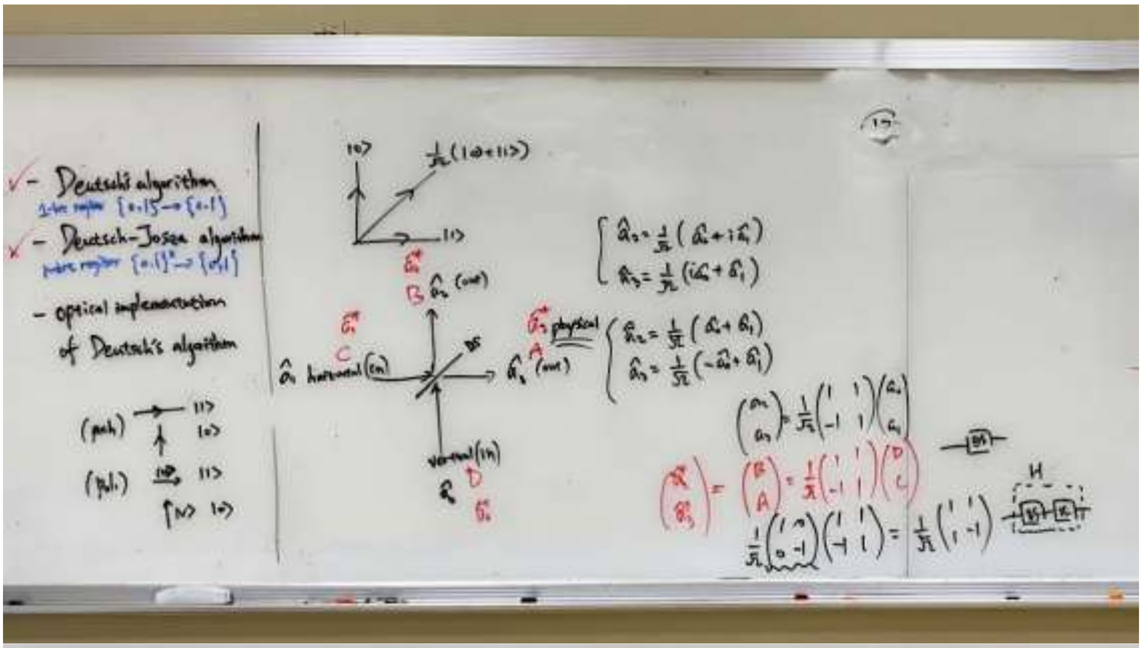

⑦
- Deutsch's algorithm
1-bit register {0,1} → {0,1}
- Deutsch-Josza algorithm
n-bit register {0,1}ⁿ → {0,1}
- optical implementation of Deutsch's algorithm
H = 1/√2 (1 1; 1 −1) (1; 0)
|0⟩
case B f(1) = f̄(0) balanced
|ψ3⟩ = (1/√2)² [|0⟩₁ (|f(0)⊕0⟩₂ − |f(0)⊕1⟩₂) + |1⟩₁ (|f̄(0)⊕0⟩₂ − |f̄(0)⊕1⟩₂)]
= (1/√2)² [(|0⟩₁ − |1⟩₁) |f(0)⟩₂ − (|0⟩₁ − |1⟩₁) |f̄(0)⟩₂]
|a⟩|0⟩ → |a⟩₁|⟩₂
H₁ → |ψ4⟩ = (1/√2)² (1/√2)⁻¹ (|1⟩₁|f(0)⟩₂ − |1⟩₁|f̄(0)⟩₂)
= 1/√2 |1⟩₁ (|f(0)⟩₂ − |f̄(0)⟩₂)

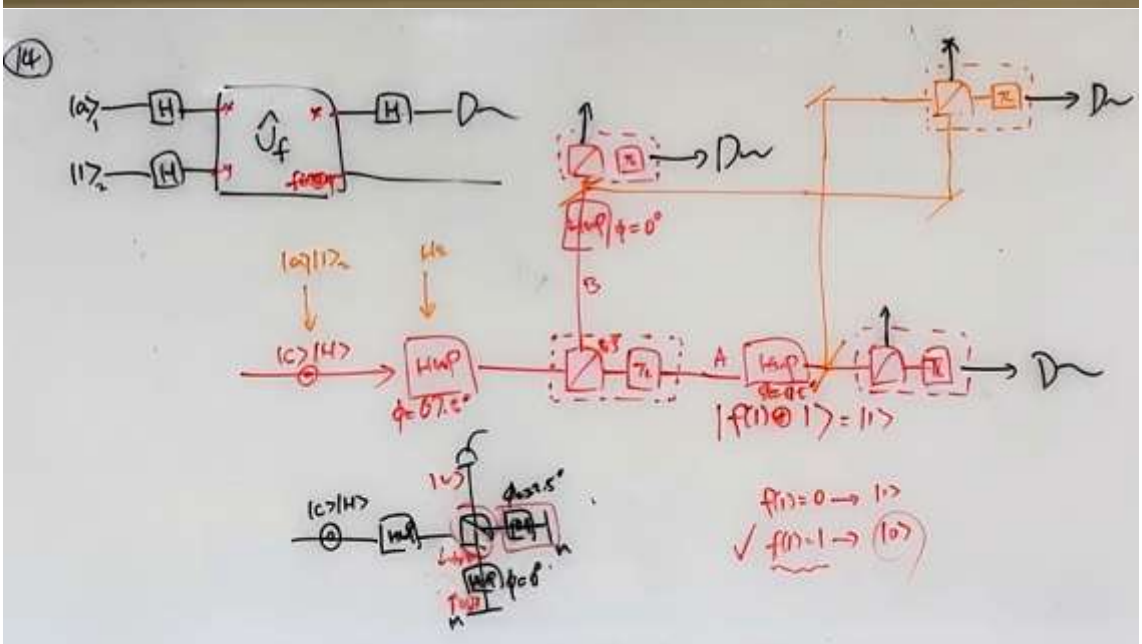

⑧
✓ - Deutsch's algorithm
1-bit register {0,1} → {0,1}
✗ - Deutsch-Josza algorithm
n-bit register {0,1}ⁿ → {0,1}
- optical implementation of Deutsch's algorithm
H = 1/√2 (1 1; 1 −1) (1; 0)
|0⟩
Deutsch-Josza construction {0,1}² → {0,1}
|ψ1⟩ |ψ2⟩ f(x) |ψ3⟩ |ψ4⟩
|a⟩₁ |b⟩₂ |1⟩₃
H H H H H
x₁ x₂ y O_f x₁ x₂ f(x)⊕y
① constant mapping f(x) = constant
② balanced mapping (half)
f(00) = f(10) = 0
f(11) = f(01) = 1
|ψ1⟩ = |0⟩₁|0⟩₂|1⟩₃ →(H₁H₂H₃) |ψ2⟩ = (1/√2)³ (|0⟩+|1⟩)(|0⟩+|1⟩)(|0⟩−|1⟩)
= (1/2)^(3/2) [|00⟩(|0⟩−|1⟩) + |01⟩(|0⟩−|1⟩) + |10⟩(|0⟩−|1⟩) + |11⟩(|0⟩−|1⟩)]

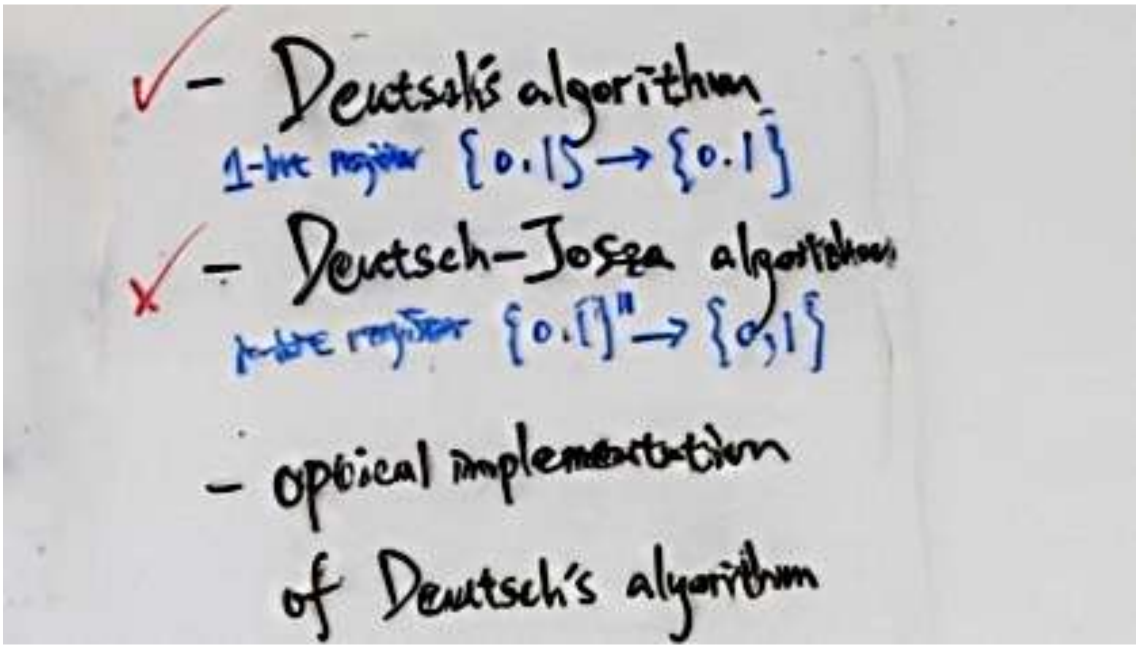

Case 1 Constant mapping
Note
|a⊕0> = |a>
|a⊕1> = |ā>
|a⊕a> = |0>
|a⊕ā> = |1>
Case 2 balanced mapping
- Deutsch's algorithm
- Deutsch-Josza algorithm
- optical implementation of Deutsch's algorithm
Summary
constant mapping
balanced mapping


(11)
- Deutsch's algorithm
1-bit register {0,1}→{0,1}
- Deutsch-Josza algorithm
- optical implementation of Deutsch's algorithm
H = 1/√2 (1 1; 1 −1)
Deutsch algorithm (optical implementation)
Classical illustration of quantum algorithm
|0⟩ |1⟩ H H U_f H
path qubit
polarization qubit
|H⟩ |1⟩
|V⟩ |0⟩
① two-qubit state
② H-gate for qubit 1
H-gate for qubit 2
③ measurement for qubit 1


(12)
optic axis
(fast axis/ordinary in birefringent material)
E_in
E_out
clockwise
counter-clockwise
E_in = |V⟩ θ=−45° E_out = |H⟩
E_in = |H⟩ θ=+45° E_out = |V⟩
X-gate (φ=45°)
|V⟩ 0° |H⟩ ±90°
Z-gate (φ=90°)
E_in = |V⟩ θ=−22.5° E_out = 1/√2 (|V⟩+|H⟩)
E_in = |H⟩ θ=−22.5° E_out = 1/√2 (|V⟩+|H⟩)
E_in = |H⟩ E_out = 1/√2 (|V⟩−|H⟩)
H-gate (φ=22.5°)
optic axis


Deutsch's algorithm
Deutsch-Josza algorithm
optical implementation
of Deutsch's algorithm
physical
horizontal
vertical


HWP
φ=67.5°
φ=0°
f(1)=0 → |1>
f(1)=1 → |0>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